студентов вузов. — М.: Аспект Пресс, 2000. — 512 с.

9. Успенский В. А. Теорема Гёделя о неполноте. — М.: Наука, 1982, — 112 с. (Популярные лекции по математики).

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ОЦЕНКА ЭФФЕКТИВНОСТИ КВАЗИОПТИМАЛЬНЫХ СТРАТЕГИЙ ДЛЯ ПЕРЕРАБОТКИ САХАРНОЙ СВЕКЛЫ

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ESTIMATING THE EFFICIENCY OF QUASI-OPTIMAL STRATEGIES FOR SUGAR BEET PROCESSING

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АННОТАЦИЯ

В статье рассматривается задача составления графика переработки сырья с неравномерными потерями производственной ценности в разных партиях. Целью данного исследования является оценка эффективности различных квазиоптимальных стратегий переработки сахарной свеклы на основе текущей информации о производственной ценности сырья. Компьютерный расчет производится с использованием реальных данных. Выход сахара, рассчитанный на основе изученных стратегий, сравнивается с абсолютным оптимумом. На основании проведенных исследований даны рекомендации по оптимизации графика переработки сахарной свеклы.

ABSTRACT

The paper considers the task of drawing up a schedule for processing raw materials with non-uniform losses of production value in different batches. The purpose of this study is to evaluate the effectiveness of various quasi-optimal sugar beet processing strategies based on current information on the production value of raw materials. A computer calculation is made using real data. The yield of sugar, calculated on the basis of the studied strategies, is compared with the absolute optimum. Based on the studies carried out, recommendations are given for optimizing the sugar beet processing schedule.

Ключевые слова: математическая модель, переработка схарной свеклы, венгерский алгоритм, квазиоптимальная стратегия

Key words: mathematical model, sugar beet processing, Hungarian algorithm, quasi-optimal strategy

1. Introduction

The task of optimizing production processes is currently of a great importance. In particular, the task of constructing an optimal schedule for processing products is relevant. Changing the processing schedule for different batches of raw materials usually does not require large expenditures, and the gain from schedule optimization is often comparable to the effect of equipment upgrades. This task is often encountered in the processing of agricultural products, for example, it takes place in the sugar industry [1-5]. Here, different batches of raw materials harvested during maturation have different production values and lose it at different rates during storage. It is required to find the best order of their processing in order to achieve the maximum yield [6-10].

The problem of constructing an optimal sequence for the processing of different batches of raw materials was considered by a number of authors. In the case of complete information about their degradation, this problem is reduced to the well-known “assignment problem” [11], which still plays a significant role in practical optimization problems [12–14]. The assignment problem is a special case of the transport problem, which is a special case of the linear programming problem. Any such problem can be solved by the simplex method, but there is a more efficient algorithm. To solve it, in 1955, Harold Kuhn developed an algorithm called the "Hungarian algorithm" [15, 16], after 2 years it was proved that it has polynomial complexity $O(n^4)$ [17]. Later [18], the Hungarian algorithm was modified to polynomial complexity $O(n^3)$. Despite the fact that it is practically impossible to form a processing strategy according to an optimal plan, it can be used to obtain an upper estimate of the objective function. In addition, other algorithms for solving this problem have been created, in particular, its important particular cases have been considered [19, 22].

In real production conditions, it is very difficult to use an abstract optimal solution, since it requires a priori knowledge of all rates of production value loss for all batches. But these rates depend on the storage conditions, in particular, on the weather conditions under which the raw material is stored. Therefore, it is impossible to know them in advance. In this case, the quasi-optimal solution becomes of a great importance. This solution only uses the current production values of the raw materials and does not guarantee the achievement of the absolute maximum output. However, it differs slightly from the optimal one in terms of production losses. Such a solution can be implemented in practice.

The purpose of this study is to evaluate the effectiveness of various quasi-optimal sugar beet processing strategies based on current information on the production value of raw materials. These strategies are simple enough to be implemented in practice. A computer calculation is made using real data (known from the experience of the Sergach sugar plant in the Nizhny Novgorod region). The yield of sugar, calculated on the basis of the studied strategies, is compared with the absolute optimum, which could be obtained by implementing the exact optimal strategy. Based on the studies carried out, recommendations are given for optimizing the sugar beet processing schedule.

The paper is organized as follows. In Section 2, the mathematical model of processing batches of raw materials is formed is given. Evaluation of some strategies compared with optimal plan and discussion of their advantages and disadvantages are presented in Section 3.

2. Statement of the problem

We suppose that there are n equal weight batches of sugar beets, numbered from 1 to n. The mass of one batch of beets is the mass that the production capacity of the enterprise can process in a certain period of time (for example, in one day). Different batches differ in a production value, i.e., the percentage of the finished product per unit of mass which corresponds to the sugar content, more exactly the percentage of sugar in beets. Let the proportion of sugar content in one kilogram of beets (sugar content) of the i th batch of beets be equal to a_i. For processing n batches of raw materials, n stages (days) are required. Each stage is indexed by means of subscript i running from 1 to n. Suppose that during storage at the j th stage of processing, the i th batch of beets loses a certain share of their production value, that is, the beets reduce their sugar content or remain unchanged, at best. Let us denote b_{ij} the coefficient of degradation which determines wilting, loss of moisture, decrease in sugar content of the i th batch of beets at the j th stage of processing. Then the production value of the ith batch of beets will change by the following way $a_i b_{i1}$ is after the first stage, $a_i b_{i1} b_{i2}$ is after the second, $a_i b_{i1} b_{i2} \dots b_{i\ n-1}$ is to the last stage of processing (unless, of course, this batch of beets is processed before this moment). It is clear that these coefficients satisfy the inequality $0 < b_{ij} \le 1$. It is assumed that during one stage of processing a given batch of beets its production value does not change. Let us number the lots of beets in descending order of their sugar content as follows

$$a_1 \ge a_2 \ge \dots \ge a_n . \quad (1)$$

If beets are processed in this order, then the yield of the finished product largely depends on the value

$$S = a_1 + a_2 b_{21} + a_3 b_{31} b_{32} + \dots + a_n b_{n1} \dots b_{n\,n-1} .$$

The yield of the finished product (sugar) depends on many values, on the percentage of dirt on the beets, the content of nitrates, damage during transportation for processing, processing temperature, and so on, but, nevertheless, the main effect on the yield of the final product has sugar content. The greater the sugar content, that is, the greater the sugar yield, all other things being equal (see, for example, the formula for the dependence of the sugar yield on the sugar content of beets in [23]). If we change the order of processing, for example, $\{3, 2, 1, 4, \dots, n\}$ instead of $\{1, 2, 3, 4, \dots, n\}$, then the yield of the finished product will take the value

$$\hat{S} = a_1 + a_2 b_{21} + a_3 b_{31} b_{32} + \dots + a_n b_{n1} \dots b_{n\,n-1}$$

Thus, the yield of the finished product depends on a given order of processing. We introduce the objective function

$$S[i(j)] = a_{i(1)} + a_{i(2)} b_{i(2)1} + a_{i(3)} b_{i(3)1} b_{i(3)2} + \dots + a_{i(n)} b_{i(n)1} \dots b_{i(n)\,n-1} , \quad (2)$$

where $i(j)$ denotes a permutation $\{i(1), i(2), i(3), \dots, n\}$ of natural numbers $\{1, 2, 3, \dots, n\}$.

If all a_i coefficients b_{ij} of degradation are known at the beginning of beet processing we can formulate the following mathematical problem: among all possible permutations of natural numbers $\{1, 2, 3, \dots, n\}$ find the permutation $i_*(j)$ at which the maximum of the function $S[i(j)]$ is achieved. It is evident that, in practice, these coefficients are unknown in advance. Therefore, it is interesting to study another orders of beets processing and to compare them with the order corresponding to the permutation $i_*(j)$.

In this paper, we will consider four various orders of beet processing which we will call the strategies.

Optimal plan. This plan corresponds to optimal order of beet processing defined by the permutation $i_*(j)$. It is important to note for any strategies, to get the yield of the finished product more then $S[i_*(j)]$ is impossible.

Greedy algorithm strategy (for more details, see [24, 25]). The processing strategy in this case is as follows. Before the start of the next stage of processing, the beet variety is determined, which by this stage has the highest sugar content. It is this variety with the highest sugar content that is sent for processing at this stage.

Strategy A. The strategy is to arrange the batches of raw materials in descending order a_i according to (1). At the beginning, the batch with the highest initial percentage of sugar content a_1 is processed, then the batches are processed in turn in descending order of initial sugar content.

Strategy B. This is absolutely arbitrary order of beet processing. At each stage a batch is taken without taking into account its sugar content. To evaluate these strategies we introduce the values for the final product yields provided by “Optimal plan”, “Greedy algorithm strategy”, “Strategy A” and “Strategy B” for some set of coefficients b_{ij} and denoted by S_0, S_g, S_A, and S_B, respectively. The relative losses generated by these strategies compared with “Optimal plan” can be expressed as

$$\overline{\mu}_g = \frac{S_0 - S_g}{S_0}, \quad \overline{\mu}_A = \frac{S_0 - S_A}{S_0},$$

$$\overline{\mu}_B = \frac{S_0 - S_B}{S_0}. \quad (3)$$

3. Results and discussion

3.1. Numerical results

In practice, when processing sugar beets not all the numerical parameters of the problem described in the previous section are known and have exact numerical values. So, if the parameters corresponding to the values of sugar content for different varieties of beets can be measured relatively accurately, then the parameters characterizing the degree of beet wilting and loss of sugar content and depending on poorly predicted weather conditions cannot be specified in advance before processing the entire harvested beet crop. In addition, in order to apply the Hungarian algorithm, it is necessary to know what the degradation factors of the batches would be before they have been processed. This can only be predicted by some empirical means, therefore, the optimal plan, in practice, generally speaking, is not achievable. The question arises, how, in this case, to correctly organize the process of beet processing. Further, it is proposed to discuss and evaluate some processing strategies.

For the numerical solution of this problem and computational experiments associated with evaluating various processing strategies, a program was written in the Python language [26]. In the computational experiments below, it is assumed that there are a total of 100 batches and sugar beet processing takes place at a hundred stages $(n = 100)$. The sugar content parameters are set randomly on the interval [0.15, 0.25] (empirical observations). The coefficients b_{ij} are set as random variables, from a segment $[\beta, 1]$, in other words, the distribution of parameters a_i and b_{ij} are obtained in accordance with the law on the uniform distribution of a random variable on the corresponding segments. Computational experiments are carried out for three different values 0.85, 0.90, and 0.95 of the parameter β. The most realistic in a practical sense, the authors consider the case of β = 0.95, the rest are present for a more detailed examination of the model. In each of figures 1 to 3, four curves are shown that represent the stepwise yield for different processing strategies. Let us describe in more detail the curves shown in figure 1, corresponding to the value of 0.85.

Figures 2 and 3 show similar curves for values equal to 0.9 and 0.95, respectively. The best optimal beet processing plan is found by the Hungarian algorithm.

The easiest way to implement in an enterprise and, as will be shown later, a quite suitable way of setting the required coefficients is to calculate their averaged values over the past few years [19-22]. Next, we will compare the effectiveness (in terms of the final result at the end of the processing process) of these strategies, as well as compare them with “Optimal plan” built using the Hungarian algorithm, and “Strategy B”, when at each stage a batch is taken without taking into account its sugar content. Naturally, the comparison will be carried out for the values of the objective function, which are obtained with different investigated processing strategies. The value of the objective function obtained during the implementation of processing strategies will be called the output.

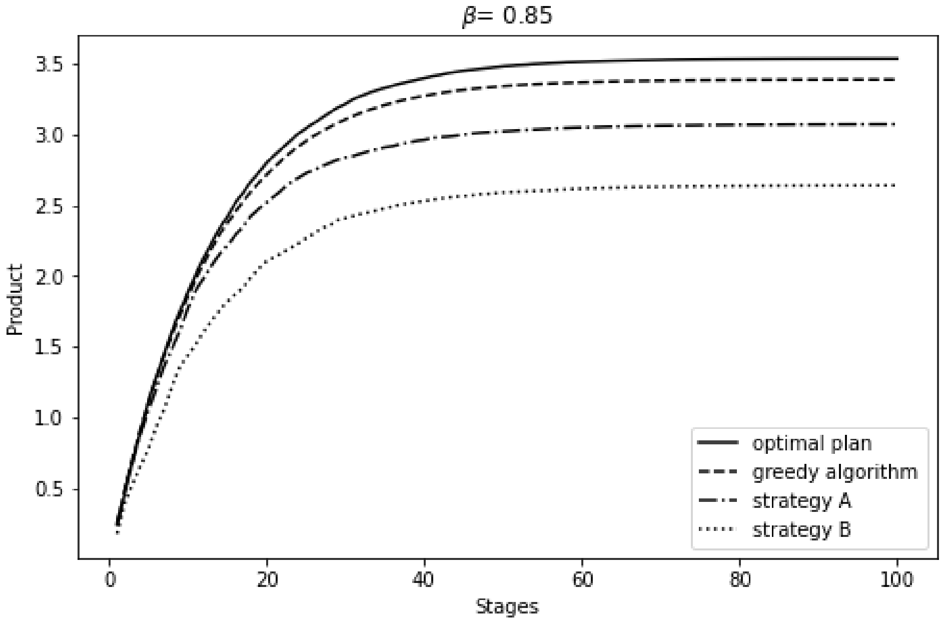

Figure1. Product yields for various processing strategies at 0.85.

In practice, this "Optimal plan" cannot be realized, however, it can serve as a kind of benchmark for evaluating other processing strategies. The "Greedy algorithm strategy" is somewhat inferior to "Optimal plan", the next in terms of efficiency in terms of the output of the finished product is "Strategy A", i.e. processing of beets in the order of successive decrease in their sugar content, and finally, the most ineffective schedule is "Strategy B", i.e. processing beet varieties in a randomly selected order.

Despite the fact that the difference between the plans may seem frivolous, it should be noted that the sugar factory processes from 3 to 6 thousand tons of raw materials per day. In fact, this is the number and you need to multiply the output shown in the figures. This descending effectiveness order of the proposed plans is intuitive, because with the optimal schedule, all available information on the degradation coefficients is used. In the greedy algorithm, information about the degradation rates at each stage is affected, but only for unprocessed batches. To implement "Strategy A", one only need to know the initial sugar content, and, finally, to carry out "Strategy B", no knowledge of degradation is needed.

Then the following computational experiment was carried out: the virtual processing of raw materials was carried out 200 times. Sets of random, uniformly distributed values of the coefficients $a_i, i=\overline{1,n}$ and uniformly distributed on the interval $[\beta,1]$of coefficients $b_{ij}, i=\overline{1,n}$, $j=\overline{1,n-1}$, n = 100, were generated. For the four different strategies presented above, the final product ratio was calculated, which was finally averaged for each processing strategy. The finished product yields are tabulated, which shows, expressed as a percentage, the relative mean "losses" obtained after applying different processing strategies compared with "Optimal plan".

To evaluate the strategies we will use the characteristics introduced in Section 2 and their average values denoted by $\langle S_0\rangle$, $\langle S_g\rangle$, $\langle S_A\rangle$ and $\langle S_b\rangle$. The relative average losses can be presented in the form

$$\mu_g=\frac{\langle S_0\rangle-\langle S_g\rangle}{\langle S_0\rangle},\quad \mu_A=\frac{\langle S_0\rangle-\langle S_A\rangle}{\langle S_0\rangle},$$

$$\mu_B=\frac{\langle S_0\rangle-\langle S_B\rangle}{\langle S_0\rangle}.$$

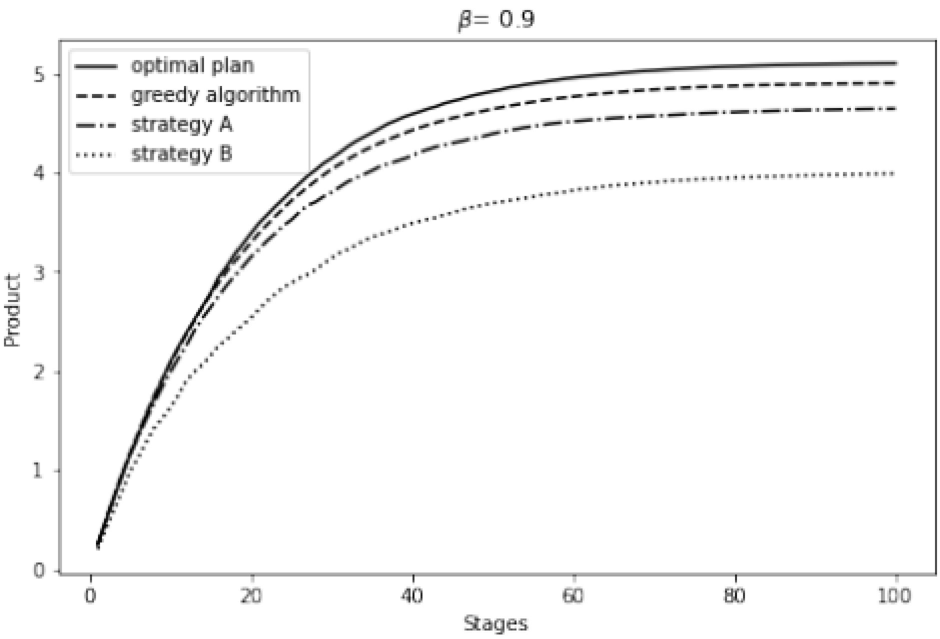

Figure 2. Product yields for various processing strategies at 0.9.

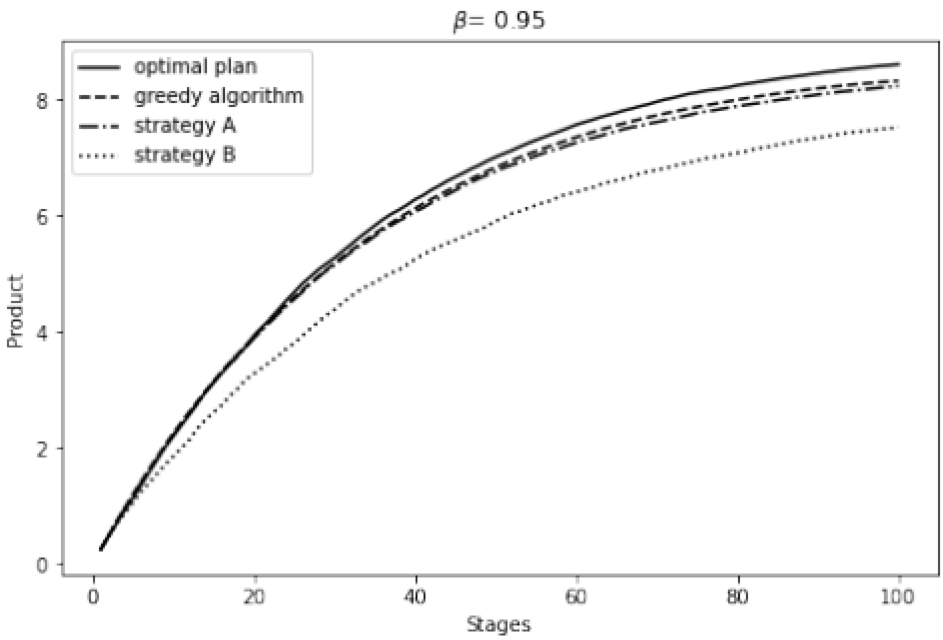

Figure 3. Product yields for various recycling strategies at 0.95.

3.2. Discussion

Let us now discuss the advantages and possible disadvantages of the above processing strategies, which can be implemented in practice. First of all, we are talking about the “Greedy algorithm strategy” and “Strategy A”. From table 1 it follows that the “Greedy algorithm strategy” provides a greater output of the finished product than “Strategy A”.

Table 1.

Relative average losses for three strategies compared with the optimal plan.

β	$\mu_g(\%)$	$\mu_A(\%)$	$\mu_B(\%)$
0.85	4.66	14.0	27.9
0.90	4.47	9.56	21.8
0.95	3.51	4.76	13.2

On the other hand, the application of the “Greedy algorithm strategy” in practice implies daily measurements of the sugar content of each of the beet varieties remaining to this moment, which can cause additional difficulties for the manufacturing plant, for example, additional costs for payment of work on carrying out measurements, or additional costs, associated with the installation of appropriate devices capable of measuring and transmitting data in automatic mode, and finally, additional costs associated with the delivery of beets with the highest sugar content from remote storage. The “Strategy A” is inferior to the “Greedy algorithm strategy” (sometimes significantly), at the same time, this strategy initially determines the order of beet processing and does not imply daily measurement of sugar content, which does not require additional costs for sugar production. Finally, the “Strategy B” based on a random choice of the beet processing order and corresponding in practice to the situation when the processing procedure is carried out according to the principle “what I want, I process this” or the batch of beets that is geographically closest to production is processed. As one can see from the figures and the table, the loss at the output of the finished product after the application of “Strategy B” in comparison with “Optimal plan”, and with the other two proposed strategies can be very significant.

4. Conclusion

This article posed the problem of the optimal processing schedule for sugar beet. An upper bound for the objective function is given. In the case when all the coefficients of the stage-by-stage degradation of beets are known, the problem is reduced to the known problem of assignments, therefore, the exact solution of the problem posed can theoretically be obtained by using the Hungarian algorithm or its varieties, but is practically not feasible in real life. In practice, all the exact coefficients of the stage-by-stage degradation of beets during the season are not known in advance. Two ways of solving the problem in conditions of uncertainty are proposed. The first, based on the use of a greedy algorithm, relies on measurements of the sugar content of each variety at the beginning of each stage. The strategy of the second (“Strategy A”) relies on a single measurement of sugar content, which is carried out before the first processing stage. As shown by a computational experiment, for large volumes of raw material batches, the losses of the greedy algorithm do not exceed 5% in comparison with the optimal one.

With an increase in the lower limit of the beet degradation coefficient to 0.95, the losses of “Strategy A” are less than 5 percent compared to the optimal processing schedule, not much worse than the losses of the ”Greedy algorithm strategy” in practice. Taking into account the resources required for measuring sugar content, “Strategy A” seems to be a more preferable strategy for processing raw materials. In addition, as shown above, “Strategy A” is optimal if the cultivation and processing of one (best) variety is carried out and the storage conditions of the batches of beets are practically identical.

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References

Skripnikov Y.G. Storage and processing of vegetables, fruits and berries. Moscow: Agropromizdat. 1986. (in Russ.)

Shirokov E.P. Technology of storage and processing of fruits and vegetables with the basics of standardization. Moscow: Agropromizdat. 1988. (in Russ.)

Manzhesov V.I. Technology of storage, processing and standardization of crop production. Saint Petersburg: Troitsky Bridge. 2010. (in Russ.)

Sumonsiri N., Barringer S. Fruits and Vegetables Processing Technologies and Applications. In book Clark S., Jung S., Lamsal B. (eds.) Food Processing: Principles and Applications. John Wiley & Sons, Ltd. 2014. \doi{10.1002/9781118846315.ch16}

Nguyen T D, Nguyen-Quang T, Venkatadri U, Diallo C and Adams M. AgriEngineering. 2021. 3. P. 519–541

Sapronov A.R.: Technology of sugar production. Moscow: Koloss. 1999. (in Russ.)

Anichin V.L. Theory and practice of production resources management in the beet sugar subcomplex of the agro-industrial complex. Belgorod: Publ. House of the BelGSHA, 2005 (in Russ.)

Kruglik S V. About optimization of technology at certain stages of sugar production // Sugar production. 2020. 4. P. 26–35 (in Russ.)

Jiao Z., Higgins A.J., Prestwidge D.B. An integrated statistical and optimization approach to increasing sugar production within a mill region. // Computers and Electronics in Agriculture. 2005. 48. P. 170-181.

Junqueira R., Morabito R. Modeling and solving a sugarcane harvest front scheduling problem. International Journal of Production Economics. 2019. 231 (1). P.150-160.

Bunday B. Fundamentals of linear programming. M.: Radio and Communications. 1989. (in Russ.)

Hydyrova G.D, Dushkina A.Y and Savina A.G. // Mathematical model of the assignment problem and the possibility of its use in making managerial decisions // Scientific notes of OrelGIET. 2014. 1(7). P. 305–310 (in Russ.)

Malyugina O.A., Chernyshova G.D. The use of the assignment problem in solving the problem of staff formation // Bulletin of the Faculty of Applied Mathematics, Computer Science and Mechanics. Voronezh: Voronezh State University. 2010. Vol. 8. P. 141-148 (in Russ.)

Martynov D.V., Dopira R.V., Abu-Abed F.N., Kordyukov R.Y., Ivanova A.V. A model and algorithmization of the assignment problem under additional constraints // Software products and systems. 2016. 2. P. 16–22 (in Russ.)

Kuhn H. The Hungarian Method for the assignment problem. // Naval Research Logistics Quarterly 1955. 2. P. 83-97.

Munkres J. Algorithms for the Assignment and Transportation Problems // Journal of the Society for Industrial and Applied Mathematics. 1957. T.5. 1.. P. 32–38

Sigal I H and Ivanova A P Introduction to applied discrete programming: models and computational algorithms. Textbook Moscow: Fizmatlit. 2002 (in Russ.)

Hopcroft J. and Karp R. An $n^{5/2}$ algorithm for maximum matchings in bipartite graphs // SIAM Journal on Computing 1973. T.2 (4). P. 225–231.

Balandin D.V., Kuzenkov O.A. Optimization of the schedule of processing of raw materials in the food industry. // Modern Engineering and Innovative technologies 2021. 17. P. 59-66. (in Russ.)

Balandin D.V., Kuznetsov Yu.A. The problem of optimizing the processing schedule of perishable agricultural products // Economic analysis: theory and practice. 2021. T.20. 11 (518). P. 2134-2150 (in Russ.)

Balandin D.V., Kuzenkov, O.A., Vildanov, V.K.: A software module for constructing an optimal schedule for processing raw materials. Modern information technologies and IT education. 2021. T.17. 2. P. 442-452 (in Russ.)

Balandin D.V, Kuzenkov O.A, Vildanov V.K. Mathematical and computer modeling and business analysis in the context of digitalization of the economy. I All-Russian scientific and practical seminar. Nizhny Novgorod, 2022. P.5-14 (in Russ.)

Kukhar V.N., Chernyavsky A.P., Chernyavskaya L.I., Mokanyuk Yu.A. Methods for assessing the technological qualities of sugar beet using indicators of potassium, sodium and α-amine nitrogen content determined in beetroot and its processed products. // Sugar. 2019. 1. P. 18-36. (in Russ.)

Asanov M.O., Baransky V.A., Racin V.V.: Discrete mathematics: Graphs, matroids, algorithms. Textbook. 2nd ed. St. Petersburg. 2010. (in Russ.).

Roughgarden T. Algorithms Illuminated. Part 3: Greedy Algorithms and Dynamic. New York: Soundlikeyourself Publishing. LLC. 2019.

Barry P. Head-First Python. Sebastopol: O’Reilly. 2016.