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ОЦЕНКА ЭФФЕКТИВНОСТИ РАВНОВЕСНОЙ СТРАТЕГИИ ДЛЯ ПЕРЕРАБОТКИ САХАРНОЙ СВЕКЛЫ

Эгамов Альберт Исмаилович
Доцент, кандидат физико-математических наук,
Нижегородский университет им. Н.И. Лобачевского, Нижний Новгород.
Гертель Арина Олеговна
Студент 4 курса ИИТММ,
Нижегородский университет им. Н.И. Лобачевского, Нижний Новгород;
Приставченко Оксана Викторовна
Магистрант 1 курса ИИТММ,
Нижегородский университет им. Н.И. Лобачевского, Нижний Новгород;

PERFORMANCE EVALUATION OF PARITY STRATEGY FOR SUGAR BEET PROCESSING

Egamov Albert Ismailovich
Docent, Candidat of phys.-mat. sciences,
Lobachevsky University of Nizhny Novgorod, Nizhny Novgorod.
Gertel Arina Olegovna
Student of 4 course IITMM,
Lobachevsky University of Nizhny Novgorod, Nizhny Novgorod.
Pristavchenko Oksana Viktorovna
Graduate student of 1 course IITMM,
Lobachevsky University of Nizhny Novgorod, Nizhny Novgorod.
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АННОТАЦИЯ

В статье рассматривается математическая модель управления процессом переработки скоропортящегося продукта – сахарной свеклы. Решается вопрос о смешивании нескольких партий во время обработки для достижения наивысшего выхода конечного продукта. В статье показано, что стратегия смешивания в любых пропорциях ни при каких обстоятельствах не увеличит оптимальную целевую функцию, полученную без смешивания различных партий. Однако, теоретически, даже смешанная стратегия может претендовать на роль квазиоптимальной. В статье предложены оценки потери целевой функции для равновесной стратегии, которая получается, если свекла из всех партий подается на вход каждый раз в равных долях.

ABSTRACT

The article considers a mathematical model for control the processing of a perishable product – sugar beet. The issue of mixing several batches during processing is being resolved to achieve the highest yield of the final product. The article shows that the mixing strategy in any proportions will under no circumstances increase the optimal target function obtained without mixing different batches. However, theoretically, even a mixed strategy can claim to be quasi-optimal. The article offers estimates of the loss of the objective function for the parity strategy, which is obtained if beets from all batches are fed to the input each time in equal parts.

Ключевые слова: математическая модель, переработка сахарной свеклы, венгерский алгоритм, квазиоптимальная стратегия, равновесная стратегия

Key words: mathematical model, sugar beet processing, Hungarian algorithm, quasi-optimal strategy, parity strategy

Introduction

The task of constructing an optimal schedule for the processing of different batches of beets, depending on their production value, was considered in a number of works by the authors [1-3]. However, when solving this problem, it should be taken into account that often One of the main problems that are solved in the organization of technological processes is the task of constructing an optimal schedule for the processing of various batches of raw materials. This task is most relevant for industrial enterprises of the agricultural sector. Here, the raw material is agricultural products, which are harvested in a limited period of maturation, and then stored for a relatively long time to ensure the smooth functioning of processing enterprises. Different batches of raw materials have different initial production value and different rate of its decline during storage, which also depends on storage conditions. The use of the best processing sequence for different batches of raw materials sometimes makes it possible to achieve a significant increase in the yield of the finished product under the same production and storage conditions. In particular, the task of optimizing the processing schedule arises in sugar production [4], where sugar beet is the raw material. The beets harvested after ripening are stored in the heap fields until processing, and different varieties of beets have different sugar content and reduce it at different rates during storage. The works of many researchers are devoted to preserving the sugar content of beets and preventing its wilting, for example, [5-11]. In order to eliminate the negative consequences of storage (freezing, rotting, etc.), producers intentionally mix different varieties of beets in different proportions when preparing raw materials for the current moment of processing. They mix lower-quality raw materials with higher-quality ones. The purpose of this work is to study the effect of mixing different beet varieties on sugar yield from the point of view of increasing the introduced sucrose and determining the optimal processing strategy to maximize the yield of the finished product taking into account this effect and to obtain an estimate of losses for the parity strategy

Materials and methods. mathematical formalization of the problem

Let sugar beet of *n* varieties, numbered from 1 to *n*, be harvested for further processing. The quantity (mass) *M* of beets of each variety is the same and is processed during one production cycle lasting a fixed period of time (one day). Accordingly, *n* processing periods are necessary to process the entire beet, individual batches of raw materials must be stored for a certain number of periods before being processed. We introduce the following notation: a_i is the sugar content (percentage of sugar content) of the *i*th beet variety, $i=\overline{1,n}$, b_{ij} is the reduction coefficient of sugar content of the *i*-th beet variety as a result of storage for the *j*-th period of time, $0<b_{ij}<1$. The sugar content of the *i*th beet variety changes as follows: $a_i b_{i1}$ is after the first period, $a_i b_{i1} b_{i2} ... b_{ik-1}$ is by the beginning of the *k*th processing period (unless, of course, it is processed before this moment). Denote p_{ij} as sugar content of the *i*th beet variety by the beginning of the *j*th processing period, then $p_{i1}=a_i$, $p_{i2}=a_i b_{i1}$, ... , $p_{in}=a_i b_{i1} \dots b_{in-1}$, $i=\overline{1,n}$. The elements p_{ij} form a square matrix of dimension $n \times n$: $\mathrm{P}=(p_{ij})$. The output of the finished product (sugar) at each processing period, other things being equal, is the greater, the greater the sugar content of the substance processed at this stage.

Let a batch of raw materials for the *j*th processing period be prepared as follows: all beet varieties are mixed in unequal proportions so that the total mass is *M*, the proportion of first grade beets is x_{1j}, the share of second grade beets is x_{2j},…, share of *i*th grade is x_{ij},…, share of *i*th grade is x_{nj},…,. Obviously, these shares must satisfy the following conditions

$$0 \le x_{ij} \le 1, \quad (1)$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ i=\overline{1,n};\ \sum_{i=1}^{n} x_{ij} = 1,\ j=\overline{1,n}. \quad (2)$$

The product yield for the entire processing time is proportional to

$$S=\sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij} x_{ij}. \quad (3)$$

The optimization problem consists in choosing the shares x_{ij}, satisfying the conditions (1), (2) under which the objective function (3) takes the maximum value $\tilde{S}$. The stated problem is a special case of the classical linear programming problem, which can be solved by the simplex algorithm, which has proven itself in economic applications.

Mixing strategies

It is clear that there is a bijection between points in a space R^{n^2} with coordinates $(x_{11}, x_{12}, ..., x_{1n}, x_{21}, x_{22}, ..., x_{2n}, x_{31}, ..., x_{nn-1}, x_{nn})$ and square matrices X of order $n \times n$. Matrices satisfying conditions (1), (2) are called bistochastic [12]. Matrices, satisfying conditions (2) and next conditions:

$$x_{ij} = 0 \text{ or } x_{ij} = 1,\ i = \overline{1, n-1},\ j = \overline{1, n-1} \quad (4)$$

are called permutation matrices. The strategy corresponding to the bistochastic matrix is called "mixed". The following Birkhoff theorem is valid.

Birkhoff theorem. *The set (1), (2) is a polyhedron in* R^{n^2} *, whose vertex coordinates are permutation matrixes [12].*

It follows from this theorem and linear programming theory that the largest (and smallest) value of the objective function (3) can always be achieved on a permutation matrix. Thus, the maximum possible result can always be achieved without mixing different batches for simultaneous processing in one production period.If variables x_{ij} satisfying the conditions (2) and (4) are considered as a solution, this problem is one of the variants of the well-known "assignment problem" [13-17].

The assignment problem is a fundamental problem of combinatorial optimization. In 1955, H. Kuhn developed the "Hungarian algorithm" [13, 14, 18] with polynomial complexity $O(n^4)$ to solve it. Subsequently, its modification was proposed, which has polynomial complexity $O(n^3)$ [19]. The Hungarian algorithm can find both the maximum and minimum value of the objective function, as well as the corresponding choice of matrix rows, that is, the extreme processing schedule.

Nevertheless, it is very difficult to use the Hungarian algorithm in practice, since it is necessary to know a priori all the degradation coefficients of the batches at all stages, even if the batches have already been processed by this stage. Therefore, the question arises: is there a certain strategy for processing batches that would be quasi-optimal and easy to apply [2, 3]. In particular, the strategy can be "mixed".

Parity schedule

3.1 Losses of parity schedule under extreme distribution of initial sugar content

Next, we consider some special cases of mixing. Suppose that the following "parity" schedule is used during processing: at each stage, all batches are processed in equal shares (the total mass is 1), that is, elements $x_{ij} = \frac{1}{n}$, $i = \overline{1, n}$, $j = \overline{1, n}$. In addition, we impose conditions on the degradation coefficients:

$$b_{ij} = b \in (0,1),\ i = \overline{1, n},\ j = \overline{1, n-1}. \quad (5)$$

We will denote the schedule losses by value $\mu = \frac{\tilde{S} - S_0}{\tilde{S}} \times 100\%$, where $\tilde{S}$, S_0 are the values of the objective functions of the optimal schedule and the parity schedule, respectively. Let us estimate the maximum possible losses of the parity schedule with respect to the optimal one, assuming that they are applied to a certain set of batches with the same parameters. Let n be an even number, we denote: $A_n = \frac{1}{n}(a_1 + ... + a_n)$, then a_k is represented as $a_k = A_n + \delta_k$, where $|\delta_k| \le \varepsilon$, $\varepsilon > 0$ is some constant and the equality $\sum_{i=1}^{n} \delta_i = 0$ is fulfilled. The objective function is

$$S = \sum_{i=1}^{n} a_i b^{i-1} = \sum_{i=1}^{n} (A_n + \delta_i) b^{i-1}.$$

Below we present such δ_i, so that this objective function would be maximum. The objective function of the parity schedule is equal to

$$S_0 = \sum_{i=1}^{n} A_n b^{i-1} = A_n \frac{1 - b^n}{1 - b}.$$

Exploring the difference between of this objective functions we get $S - S_0 = \sum_{i=1}^{n} \delta_i b^{i-1}$, taking into account equality $\sum_{i=1}^{n} \delta_i = 0$ and inequalities $|\delta_i| \le \varepsilon$. The task is transformed to the following: *Find max* $\sum_{i=1}^{n} \delta_i b^{i-1}$, *under conditions* $\sum_{i=1}^{n} \delta_i = 0$ and $|\delta_i| \le \varepsilon$. Solving it, we get, that first $n/2$ variables $\delta_i = \varepsilon$, $i = \overline{1, n/2}$, other $n/2$ variables $\delta_i = -\varepsilon$, $i = \overline{n/2+1, n}$. Then

$$\tilde{S} - S_0 = \sum_{i=1}^{n/2} \varepsilon b^{i-1} - \sum_{i=n/2+1}^{n} \varepsilon b^{i-1} = \frac{\varepsilon(1-b^{n/2})^2}{1-b}.$$

For a given δ_i, if we will accept $n = 50$, $\varepsilon = 0.1$, A_n=0.2, b=0.95, losses are equal $\mu = \frac{\tilde{S} - S_0}{\tilde{S}} = \frac{1.0443}{1.0443 + 3.6924} \approx 22\%$.

Having presented the solution of the problem in a similar way, we note that, in fact, instead of 50 different parties, there are only two of them in this solution. It turned out 2 sets of 25 batches with initial sugar content $A_n + \varepsilon$ and $A_n - \varepsilon$. Note that the more ε, the more μ.

However, it is easy to see that the functions $\tilde{S}$, S_0 are continuous and even uniformly continuous with respect to their parameters; therefore, by slightly varying the initial sugar content within the tolerance, 50 different batches can be obtained with a losses of at least 20 percent.

1.2Losses of parity schedule with uniform distribution of initial sugar content

Consider another example, here the initial sugar content of the batches is "evenly distributed" on the allowable interval, that is, it is an arithmetic progression located in the allowable interval $[0.15, 0.25]$: $a_1 = 0.25,$ $a_{k-1} = a_k + h,$ $k = \overline{2, n}$, $a_n = 0.15,$ $\Delta a = 0.1,$ that is, the difference between close parameters a_k is equal to $h = \frac{a_1 - a_n}{n-1} = \frac{0.1}{n-1}$, $A_n = 0.2$, $a_k = a_1 - (k-1)h$, $k = \overline{1, n}$, that is $a_k = A_n + 0.5\Delta a - (k-1)h.$ The Equalities (5) hold. In this case, according to the rearrangement inequality [20], the optimal objective function is equal to

$$\tilde{S} = \sum_{i=1}^{n} a_i b^{i-1} = \sum_{i=1}^{n} (A_n + 0.5\Delta a - (i-1)h) b^{i-1}.$$

According to (8) the difference between of this objective functions:

$$\tilde{S} - S_0 = 0.5 \cdot \Delta a \cdot \sum_{i=1}^{n} b^{i-1} - h \sum_{i=1}^{n} (i-1) b^{i-1}. \qquad (6)$$

We will use information from mathematical analysis. Notice, that $f(z) = \sum_{i=1}^{n} z^{i-1} = \frac{1-z^n}{1-z}$. Its derivative is

$$f'(z) = \sum_{i=1}^{n} (i-1) z^{i-2} = \frac{d}{dz}\left(\frac{1-z^n}{1-z}\right) = \frac{-nz^{n-1}(1-z) + (1-z^n)}{(1-z)^2}.$$

Therefore expression (6) will be rewritten as

$$\tilde{S} - S_0 = 0.5\Delta a \frac{1-b^n}{1-b} - hb\frac{-nb^{n-1}(1-b)+(1-b^n)}{(1-b)^2} =$$

$$= 0.5\Delta a \frac{1-b^n}{1-b} - \frac{hb}{(1-b)^2}\cdot((n-1)b^n - nb^{n-1} + 1).$$

If we will accept $n = 50$, $\varepsilon = 0.1$, A_n=0.2, b=0.95, the losses of the parity schedule will be

$$\frac{\tilde{S} - S_0}{\tilde{S}} = \frac{0.2685}{3.6924 + 0.2685} \approx 6.78\,\%.$$

2.Conclusion

With the same degradation coefficient, the parity schedule gives a greater deviation from the optimal target function, the greater the dispersion of the initial sugar content. An attempt is made to use the parity strategy as a quasi-optimal plan. It is clear that with a small variance of the initial sugar content, due to the uniform continuity of the objective function with respect to it's the parity strategy, it will be quasi-optimal. With a large dispersion of parameter a, the use of parity strategy is not desirable.

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